NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

# WARTIME REPORT

ORIGINALLY ISSUED
September 1941 as
Advance Restricted Report

## DYNAMIC STRESS CALCULATIONS FOR TWO AIRPLANES IN VARIOUS GUSTS

By Harold B. Pierce

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

NACA

WASHINGTON

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## SUMMARY

A series of calculations was made to determine the probable dynamic wing stress of two large airplanes in atmospheric gusts. These calculations were undertaken concurrently with a more general investigation, still incomplete, from which it appears at the present time that the calculated stress agrees well with measurements on flexible wing models in the gust tunnel.

The results of these special calculations indicate that in both isolated and repeated gusts of probable occurrence the dynamic overstress is about 10 percent when referred to the present static design standard.

## INTRODUCTION

The possibility of dynamic overstress in airplane wings upon encountering atmospheric gusts has been the subject of a number of investigations. The formulas and methods resulting from these investigations when applied to specific cases as a rule showed overstress; whereas preliminary tests of a flexible-wing model in the gust tunnel consistently showed understress. In view of this conflict it was felt that a further development of the theory and more comprehensive tests should be undertaken.

This work has been carried out to an advanced stage, and reasonably good agreement has been found between the theoretical and experimental results so far as this part of the work has been carried. The analysis of actual cases has, however, been largely confined to obsolescent designs in single gusts, in which cases understress rather than overstress was the outstanding result. In view of the rapidly changing trend in design, it was felt desirable to apply the results of the investigation to two airplanes currently in the design stage of development, and to extend the analysis to the effect of repeated gusts. This special analysis disclosed possibility of serious overstress; accordingly, the results are presented herewith for the information of those concerned. A more comprehensive report on the complete investigation is intended to follow.

For the purpose of this report, the airplane designs in question will be designated model A and model B.

## METHOD

Inasmuch as a more detailed discussion of the theory will be presented in a later report, only a brief outline will be given here.

An airplane flying through the air may be considered in the spanwise direction as a beam of nonuniform cross section supported on a yielding foundation. Since a rigid solution of the problem of dynamic stress for such a beam is impractical, as pointed out by Küssner in reference 1, an equivalent wing and spring system for an airplane was assumed as indicated in figure 1. The motion of the upper wing is adjusted to be that of the wing tip motion of the original airplane and the motion of the lower wing and fuselage is the motion of the fuselage of the original airplane under shear load from its wing. The equations of motion for this system under the influence of a single gust of the type shown in figure 2(a) are as follows:

$$M_{w_e}D^2\delta_w + \lambda_{w_e}D\delta_w + K(\delta_w - \delta_f) = A_{w_e}te^{-bt} \qquad (1)$$

$$M_{f_e}D^2\delta_f + \lambda_{f_e}D\delta_f - K(\delta_w - \delta_f) = A_{f_e}te^{-bt} \qquad (2)$$

where the subscript $w$ refers to wing quantities and the subscript $f$ refers to fuselage quantities, and

| | |
|---|---|
| $t$ | time |
| $M_{w_e}$ | equivalent mass of wing |
| $M_{f_e}$ | equivalent mass of fuselage |
| $D$ | differential operator |
| $\delta_w$ | space position of wing (fig. 1) |
| $\delta_f$ | space position of fuselage (fig. 1) |

$K$ equivalent spring constant based on wing frequency and equivalent wing weight

$A_{w_e}te^{-bt}$ and $A_{f_e}te^{-bt}$ forcing functions or air loads on wing and fuselage

$\lambda$ damping coefficient $= \lambda_{w_e} + \lambda_{f_e} =$ (effective damping factor) $m\frac{\rho}{2}SV$

$m$ slope of lift curve, per radian

$\rho$ air density

$S$ wing area

$V$ airspeed

The equivalent mass of the wing, $M_{w_e}$, is the mass which, if placed at the wing tip of a weightless beam, would give the same deflection under unit acceleration as the distributed mass of the wing. In this case it is $M_{w_e} = \sum_{0}^{\bar{x}}\left[\Delta M\left(\frac{x}{\bar{x}}\right)^{3}\right]$ as shown in figure 1.

The equivalent mass of the fuselage, $M_{f_e}$, is equal to the mass of the fuselage plus the actual mass of the wing minus $M_{w_e}$.

In the absence of knowledge of the actual deflection curve of a wing, an assumption must be made for the purpose of computing the damping coefficient. If, as will be assumed here, the deflection at any point is proportional to the square of the distance from the root, the damping of the wing tip motion is equal to the damping of the whole wing times the ratio of the deflection of the mean vertical velocity position to the tip deflection, or $\lambda_{w_e} = 1/3\lambda$. The effective damping factor for the range of wing frequencies of interest is taken as 0.75 from the results of a preliminary analytical and experimental investigation.

The total air load on the wing may be considered as consisting of two components, one of which results in bending deflection at the tip and the other of which results in a shear force at the wing root. The division of the air load into these components is accomplished in much the same manner as the division of mass of the wing. The shape of the air load on the wing was assumed in the cases of both airplanes investigated to be similar to that on an average tapered wing and it was divided so that $A_{w_e}te^{-bt}$ applied at the wing tip gave the same deflection as the total distributed air load. For the cases under investigation it can be shown that $A_{w_e} = 0.25A$ where

$$A = \frac{\Delta n \times W}{te^{-bt}} \text{ at } bt = 1 \tag{3}$$

W total weight of the airplane

$b = \frac{1}{t}$ at bt = 1 or the maximum value of the function $te^{-bt}$

Since $A_{w_e}$ and $A_{f_e}$ appear as multiplying factors for the equations in their solved form, $\Delta n$ may be any arbitrary load factor increment; $\Delta n$ is the load factor increment that the airplane would experience if it had no vertical motion as it traveled through the gust.

The function $te^{-bt}$ was taken as most closely representing the shape of the time history of air load on the wings in a gust as indicated by tests in the gust tunnel.

Equations (1) and (2) were put in operational form, combined, and solved to give the following results:

$$\delta_f = e^{-R_1t}\left(c_1 \cos R_2t + c_2 \sin R_2t\right) + c_3e^{-R_3t} + c_4$$
$$+ K_1te^{-bt} + K_2e^{-bt} \tag{4}$$

$$\delta_w = \frac{M_{f_e}}{K}D^2\delta_f + \frac{\lambda_{f_e}}{K}D\delta_f + \delta_f - \frac{A_{f_e}}{K}te^{-bt} \tag{5}$$

An expression for the normal acceleration of the airplane when the wings are held rigid may be derived by letting $(\delta_w - \delta_f)$ equal zero, so that $D^2\delta_f = D^2\delta_w$ and $D\delta_f = D\delta_w$ in equations (1) and (2). With this restriction equations (1) and (2) may be combined to become:

$$\left(M_{w_e} + M_{f_e}\right) D^2\delta_w + \left(\lambda_{w_e} + \lambda_{f_e}\right) D\delta_w = \left(A_{w_e} + A_{f_e}\right) te^{-bt} \tag{6}$$

where $D^2\delta_w = \Delta n_r$, the acceleration of the rigid airplane.



The gust gradient distance $H_1$ is determined by the time interval from the start to the peak of the $\Delta n_r$ curve. Gust-tunnel tests of rigid wing models substantiate this method since the normal acceleration on the model reaches its maximum value at the same time the gust velocity of a linear gust gradient reaches its maximum value.

For the purpose of these calculations the wing deflection is assumed to be proportional to the wing stress. The dynamic stress may therefore be given in terms of the ratio of the dynamic wing tip deflection $(\delta_w - \delta_f)$, which is designated $\delta_d$, to the static deflection $\delta_{st}$ under the same condition of load. In this case $\delta_{st}$ may be determined from the equation

$$\delta_{st} = \frac{\Delta n_{r_{max}} \times 0.25(W_{tot})}{K} - \frac{\Delta n_{r_{max}} W_{w_e}}{K} \tag{7}$$

The time history of reactions calculated for a single gust are basic curves to which the principle of superposition may be applied to determine the reactions for repeated gusts. This method is substantiated in reference 2 where an analogous problem in electricity is presented.

An alternative method of determining the reactions for a repeated gust would be to apply a suitable repeating force function to the equation. This method would be tedious, however, since each gust combination would have to be calculated separately; whereas, by superposition, three basic sets of curves representing different probable gust gradient distances may be used to give an almost unlimited number of combinations of two gusts.

There is no direct mathematical connection between the gust velocity distribution and the forcing function since the forcing function was determined by inspection of gust-tunnel records. The distribution must, however, approximate that shown in figure 2(a). The gust velocity distribution for a repeated gust therefore may be obtained by adding together two gusts of this type as shown in figure 2(b). The result of this addition may be of the type shown in figure 2(c) or 2(d).

Although the effect of airplane stability is not taken into account directly, it is felt that the form of the forcing function in some degree takes into account this effect. However, it is believed advisable, because of the limitations introduced by lack of direct consideration of the pitching motion, to limit the number of superpositions to two gusts.

## CALCULATIONS AND RESULTS

The conditions and basic constants used for the calculation of the reactions of model A and model B are given in table I.

In the practical application of dynamic stress calculations, it is essential that the true nature of the force causing the stress be known. In the present instance, this is equivalent to saying that the gradient distance of the gust be known, since it is this distance that determines the nature of the forcing function. Fortunately, it can be stated that the most probable gradient distance associated with the largest gust accelerations is about 10 chord lengths and that the severity of the acceleration falls off rapidly with a departure of the gradient distance from this value. This fact has been well demonstrated by extensive flight tests on airplanes varying in size from about 1500 pounds to 65,000 pounds, and can be demonstrated by physical considerations. In order to obtain results of practical interest, therefore, the calculations for models A and B have been carried out for gusts having a gradient distance of 10 chord lengths. In addition to these calculations, however, further calculations have been made for gradient distances of about 4 chord lengths and of about 20 chord lengths in order to extend the range of the analysis.

Time history curves of the reactions of models A and B to single gusts having gradient distances of about 4, 10, and 20 chord lengths are given in figures 3 to 5.

Sample time history curves for a repeated gust composed of two equal and opposite 10 chord-length gusts arranged relative to one another to give maximum negative wing deflection are given in figures 6 through 8.

Figures 9 through 11 show for single gusts the variation with gradient distance of the dynamic stress ratio or ratio of maximum dynamic wing deflection to the static deflection. They also show the variation of the ratios of maximum wing-tip and fuselage accelerations to the maximum accelerations for the case of the rigid wing.

Since the interest is primarily in dynamic stress rather than in accelerations, the superposition of the calculated basic curves was done with a view to determining the maximum overstress from the combination of the reactions of two gusts. It was found that maximum values of dynamic stress ratio occurred when the repeat gust was of a negative sense in relation to the first gust and that the sequence period or distance $H_3$ (see fig. 2) had a pronounced effect on the result. The maximum value of $\Delta n_r$ occurring in the whole sequence was used to determine the static deflection. Table II presents selected cases which were the most serious of a number of combinations examined. In this table the results refer to repeat gusts having the same values of $\Delta n$ as the initial gusts. It should be noted that the true gust velocities of the initial and repeat gusts for any condition are therefore the same, and independent of the value of $H$ since, it will be remembered, $\Delta n$ is the load factor increment that the airplane would experience if it had no vertical motion as it traveled through the gust.

## DISCUSSION OF RESULTS

The results of the calculations for model A in a single gust (figs. 9 and 10) and for model B (fig. 11) show that the dynamic stress in all cases increases from an understress for a gust of long gradient distance to moderate to high overstress for a short gradient distance. The general shape of the curves seems to indicate that even higher overstress would occur in sharper gusts than those examined. However, the lag in development of lift in sharper gusts would preclude such a result, since even for an infinitely sharp gust, the forcing function would have a character similar to that for 4 chord lengths.

Differences in the values of dynamic stress between the two conditions of model A would indicate that the change in forward speed and weight condition of the airplane has a pronounced effect. The more general analysis previously mentioned has shown that, in general, the dynamic stress increases a small amount with increase in velocity but that the effect is small. The change in weight and weight distribution produces a substantial change in dynamic stress and, in general, a change in these qualities of such nature as to reduce the wing frequency will result in an increase in the value of the dynamic stress.

When the basic curves in this paper are superposed to obtain the maximum overstress from their combination, it is seen from the results presented in table II that there is no definite correlation between the effect of gradient distance of the first and second gusts and the distance between them $H_3$. This lack of correlation results from the influence of certain other factors, such as the relation between the time to peak acceleration and the period of wing vibration, which complicate the problem when the reactions to one gust are superposed on those to another gust.

Examination of the values in table II shows that substantial overstress exists for all the combinations of gusts presented and that the addition of a short gradient gust produces the largest value. As indicated in the preceding discussion, the value of overstress becomes large in all cases as the gradient distance of the gust is decreased. However, before an estimate can be made as to whether the overstress on the airplanes in question will be serious, it will be necessary to consider the effect of the intensity and size of gusts and their spatial distribution in the atmosphere.

## APPLICATION OF RESULTS

It should be emphasized that any process chosen at this time to appraise the significance of the dynamic stress calculations with respect to practical design questions must be viewed with considerable suspicion. The subject is entirely too broad and involved to be given adequate treatment here although, in order to preclude improper application of the results, a brief discussion seems necessary at this point.

The present design critierions for gust loads are based on thousands of hours of acceleration-air speed data obtained with V-G recorders installed on transport airplanes. The analysis of these data has indicated that a reasonable value of the effective gust velocity to be used for design purposes is 30 feet per second. As previously mentioned, the most probable gradient distance associated with this gust velocity has been found from separate investigations, such as that reported in reference 3, to be about 10 chord lengths. As shown further in these investigations, the gust velocity measured on a given airplane increases from a negligible value to a maximum as the gradient distance is increased from 0 to 10 chord lengths, but with further increase in gradient distance the measured gust velocity tends to fall off.

In view of these facts it is felt that the overstress indicated in the short single gusts may be disregarded for the reason that the total stress will be less than the stress for the more important gusts even without overstress.

It is further felt, however, that the amount of overstress shown for the airplanes in question in single gusts when the gradient distance is 10 chord lengths (figs. 10 and 11) should be added to the design stress calculated on the usual basis.

So far as repeated gusts are concerned, there are no data available concerning the size, intensity, and probability of occurrence of such gusts, since no practical method of analysis of available records to obtain such information has been found to date. It is necessary, therefore, to appraise the probable overstress in repeated gusts on the basis of measurements of the structure and the frequency of single gusts. For this purpose let it be assumed, for the sake of illustration, that any airplane will, during its life, encounter only one single gust of the size and intensity corresponding to the present design effective gust. With this assumption, it follows that each gust of any probable combination of repeated gusts must have less intensity than the single design gust. Such a restrictive assumption is not really necessary, as the same argument applies in a relative or qualitative sense if the design effective gust is encountered several times during the airplane life.

In order to determine the reduced value of the repeat gusts, recourse was had to gust measurements made on a number of airplanes. Reference 3 presents data of the type used for this purpose. These data were applied on the assumption that the maximum intensity of the three gusts next in intensity to the maximum gust appearing in the data would bear approximately the same ratio to the intensity of this maximum gust as the intensities of the probable repeated gusts would bear to the single design gust. With this assumption the ratios were found from the several sets of data to be 0.75, 0.66, and 0.61. Thus the intensity of the individual gusts in a repeat combination may, depending on the data used, be

$$U_{rpt} = 0.75\ U_{max}, \quad \text{etc.}$$

Referring to figure 2, this means that the repeat gust combination would appear as in (c) with the value of $U_{max}$ equal to 0.75 (etc.) times the value of $U_{1_{max}}$ appearing in (a).

The ratio of the maximum stress experienced in the repeat gust sequence to the static stress usually calculated for the standard design gust may thus be obtained by multiplying the ratio of the maximum dynamic stress in the sequence to the static stress in the first phase of the repeat combination by the ratio of $U_{rpt}$

to $U_{max}$. Note that this process differs from that employed in obtaining the dynamic stress ratios of table II in which the maximum value of $\Delta n_r$ in the whole sequence was used to determine the static deflection.

Confining the remainder of the analysis to repeat gusts having values of $H_1$ and $H_2$ of 10 chord lengths, but utilizing any value of $H_3$, cases may be selected out of table II for further examination. The cases selected are those marked with an asterisk in the first column of table II. The maximum dynamic stress in these cases, referred to the static stress in the first phase, are as follows:

| Model | Condition | $\delta_d/\delta_{st}$ |
|---|---|---|
| A | I | 1.58 |
| A | II | 1.64 |
| B | I | 1.62 |

These values, multiplied by the repeat-gust intensity ratios 0.75, 0.66, and 0.61, previously explained, have been plotted in figure 12 against the intensity ratio. On the whole, figure 12 indicates that some dynamic overstress is to be expected in repeat gusts relative to the present design standard. Taking a mean value, the overstress is about 10 percent, or about the same amount found for the single gust. In view of these results, it is felt that the design strength for these airplanes in the gust condition should be increased about 10 percent.

## CONCLUSIONS

The analysis of the results of the calculations for models A and B indicates:

1. The dynamic stress in the airplane wings from encountering gusts in the atmosphere increases as gradient distance decreases.

2. For the established important gradient distance of 10 chord lengths, the overstress in a single gust is about 10 percent.

3. When probable types of repeated gust are encountered, the airplane wings may be stressed about 0 to 20 percent beyond the stress allowed by the present design gust load factor.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va.

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## REFERENCES

1. Küssner, Hans Georg: Stresses Produced in Airplane Wings by Gusts. T.M. No. 654, NACA, 1932.

2. Berg, Ernst Julius: Heaviside's Operational Calculus. McGraw-Hill Book Co., Inc., 1936, pp. 42-43.

3. Donely, Philip: Effective Gust Structure at Low Altitudes as Determined from the Reactions of an Airplane. Rep. No. 692, NACA, 1940.

TABLE I.- CONDITIONS AND BASIC CONSTANTS OF MODEL A AND MODEL B

Model A

| | Condition I | Condition II |
|---|---|---|
| Weight, lb | 62,500 | 102,000 |
| Wing area, sq ft | 1,826 | 1,826 |
| Span, ft | 140 | 140 |
| Mean geometric chord, ft | 13.04 | 13.04 |
| Natural wing frequency, cps | 2.50 | 1.43 |
| Slope of lift curve, per radian | 4.93 | 4.93 |
| Forward velocity, mph | 190 | 160 |
| $W_{w_e}$, lb | 1,674.4 | 4,974.2 |
| $W_{f_e}$, lb | 60,882.6 | 97,025.8 |
| Spring constant, lb per ft | 12,405.75 | 12,405.75 |
| $\Delta n$ | 2 | 2 |

Model B

| | Condition I |
|---|---|
| Weight, lb | 100,000 |
| Wing area, sq ft | 1,710 |
| Span, ft | 140 |
| Mean geometric chord, ft | 12.21 |
| Natural wing frequency, cps | 2.45 |
| Slope of lift curve, per radian | 5.04 |
| Forward velocity, mph | 260 |
| $W_{w_e}$, lb | 3,425.4 |
| $W_{f_e}$, lb | 96,574.6 |
| Spring constant, lb per ft | 25,233.1 |
| $\Delta n$ | 2 |

TABLE II.- MOST SERIOUS VALUES OF OVERSTRESS FROM ADDING THE REACTIONS OF TWO GUSTS

| $H_1$ chord lengths | $H_2$ chord lengths | $H_3$ chord lengths | Maximum $\delta_d/\delta_{st}$ |
|---|---|---|---|
| **Model A, condition I** | | | |
| 4.26 | 4.26 | 14.50 | 1.20 |
| 4.26 | 10.25 | 11.96 | 1.09 |
| 4.26 | 19.53 | 5.13 | 1.09 |
| 10.25 | 4.26 | 35.04 | 1.31 |
| *10.25 | 10.25 | 32.48 | 1.21 |
| 10.25 | 19.53 | 25.64 | 1.20 |
| 19.53 | 4.26 | 69.22 | 1.42 |
| 19.53 | 10.25 | 66.66 | 1.34 |
| 19.53 | 19.53 | 59.82 | 1.39 |
| **Model A, condition II** | | | |
| 3.96 | 3.96 | 15.47 | 1.47 |
| 3.96 | 10.08 | 13.31 | 1.35 |
| 3.96 | 20.15 | 11.87 | 1.35 |
| 10.08 | 3.96 | 35.98 | 1.47 |
| *10.08 | 10.08 | 33.83 | 1.26 |
| 10.08 | 20.15 | 32.39 | 1.11 |
| 20.15 | 3.96 | 73.41 | 1.56 |
| 20.15 | 10.08 | 71.25 | 1.36 |
| 20.15 | 20.15 | 69.81 | 1.23 |
| **Model B, condition I** | | | |
| 3.75 | 3.75 | 16.23 | 1.40 |
| 3.75 | 9.99 | 13.74 | 1.26 |
| 3.75 | 19.98 | 3.75 | 1.26 |
| 9.99 | 3.75 | 39.94 | 1.35 |
| *9.99 | 9.99 | 37.46 | 1.25 |
| 9.99 | 19.98 | 27.47 | 1.08 |
| 19.98 | 3.75 | 72.43 | 1.44 |
| 19.98 | 9.99 | 69.93 | 1.34 |
| 19.98 | 19.98 | 59.94 | 1.20 |

*Significance given in text.

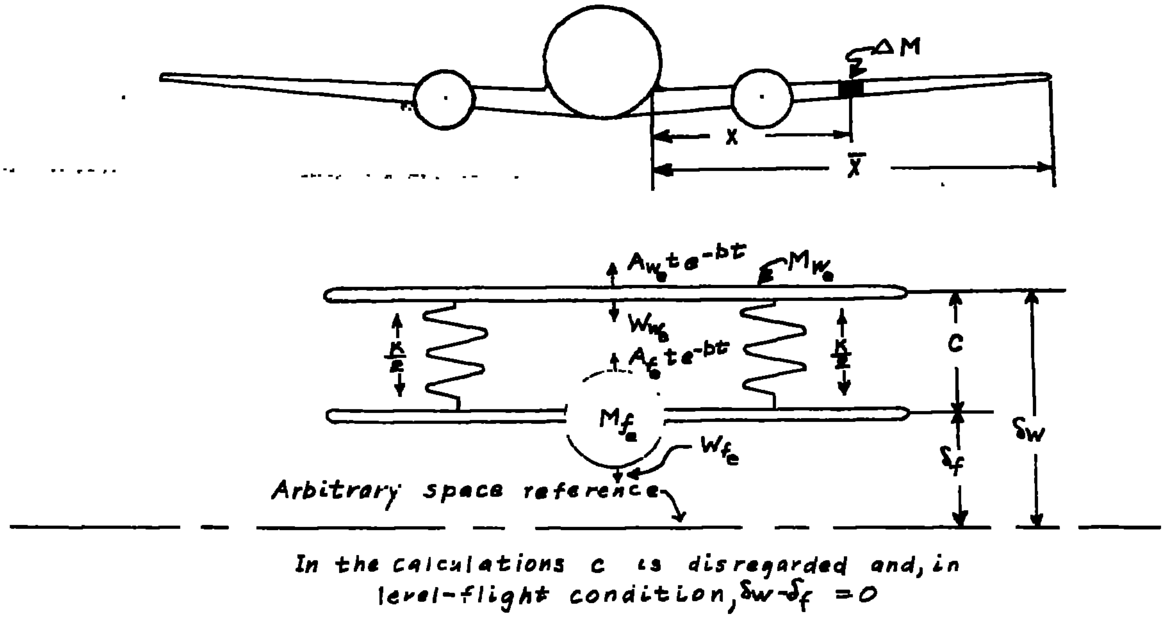



Figure 1.- Equivalent wing and spring system.

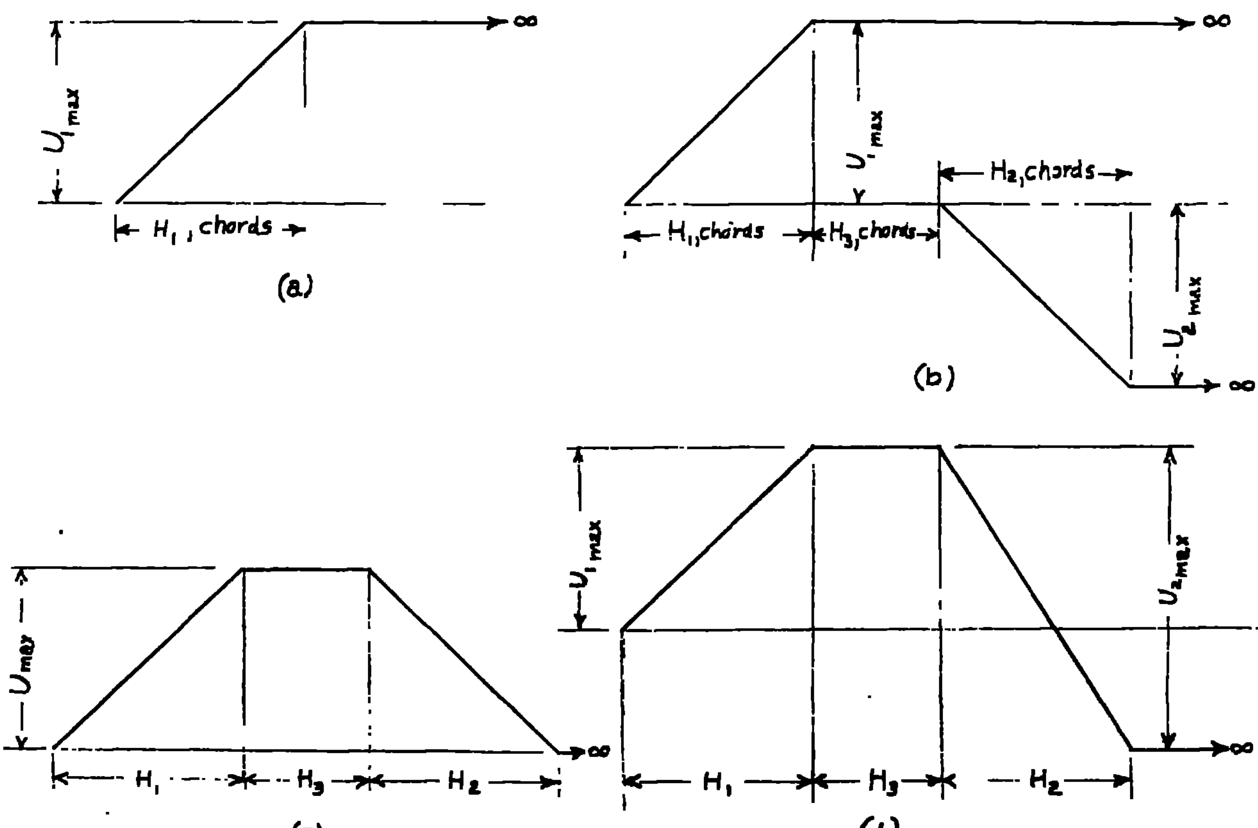
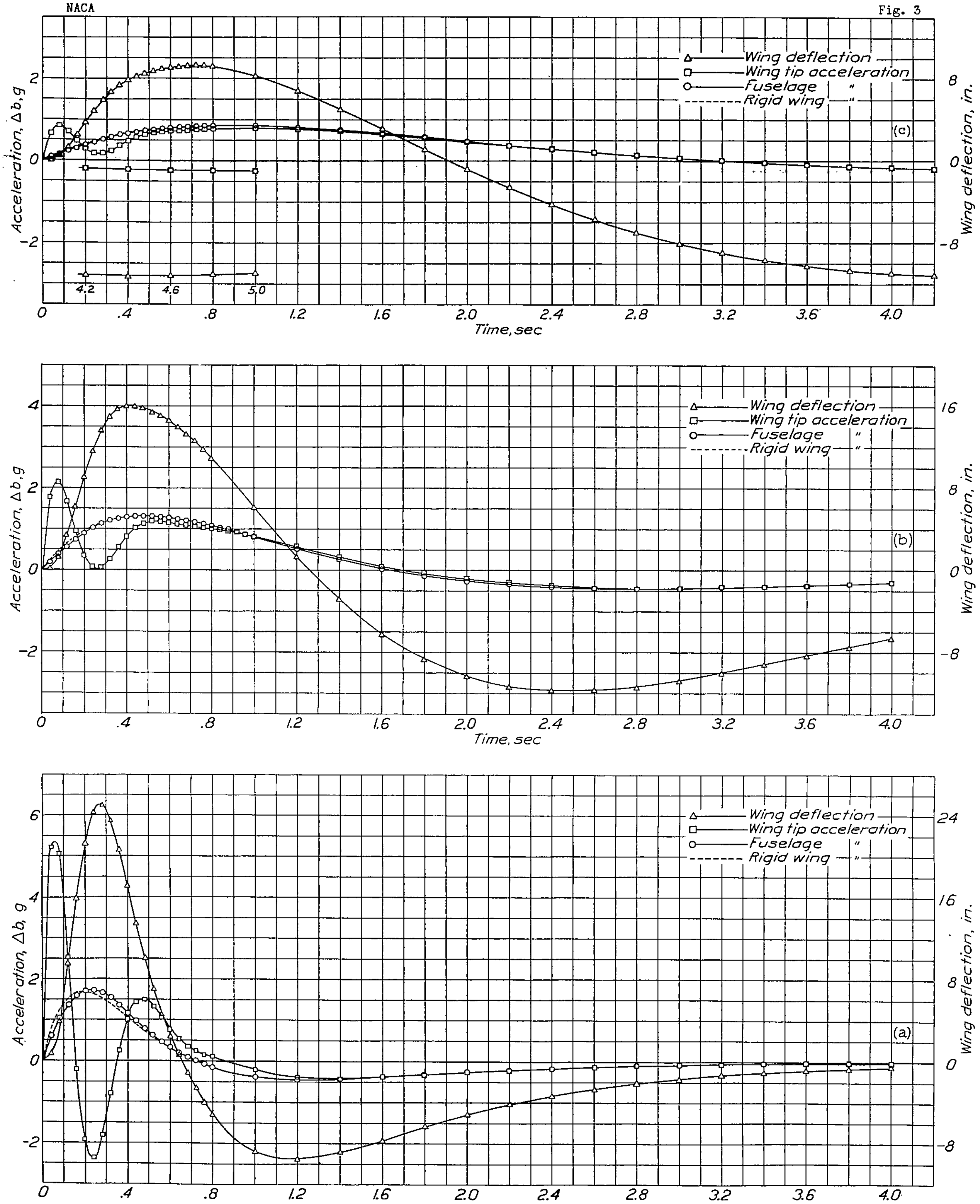
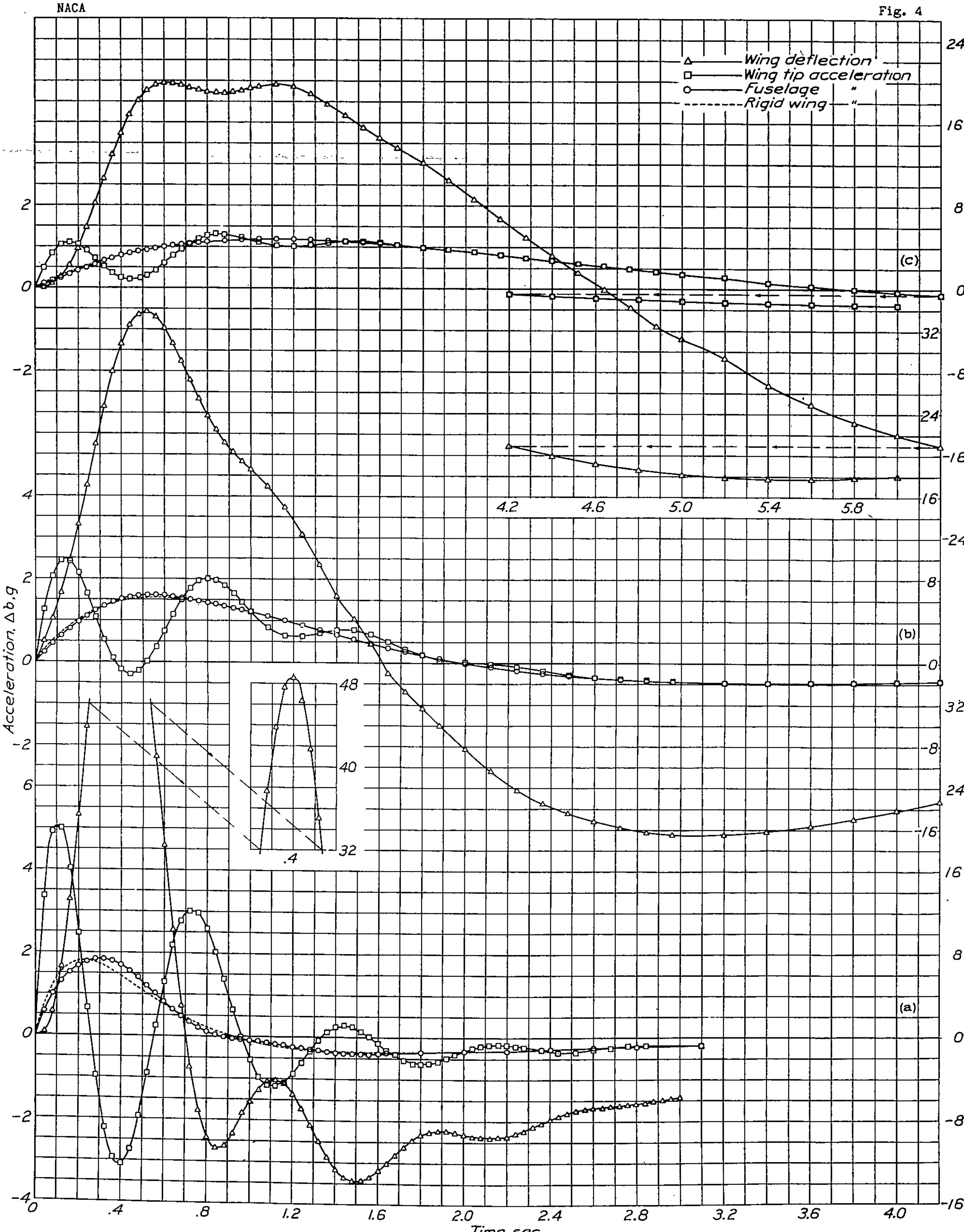



Figure 2. - Addition of gusts.



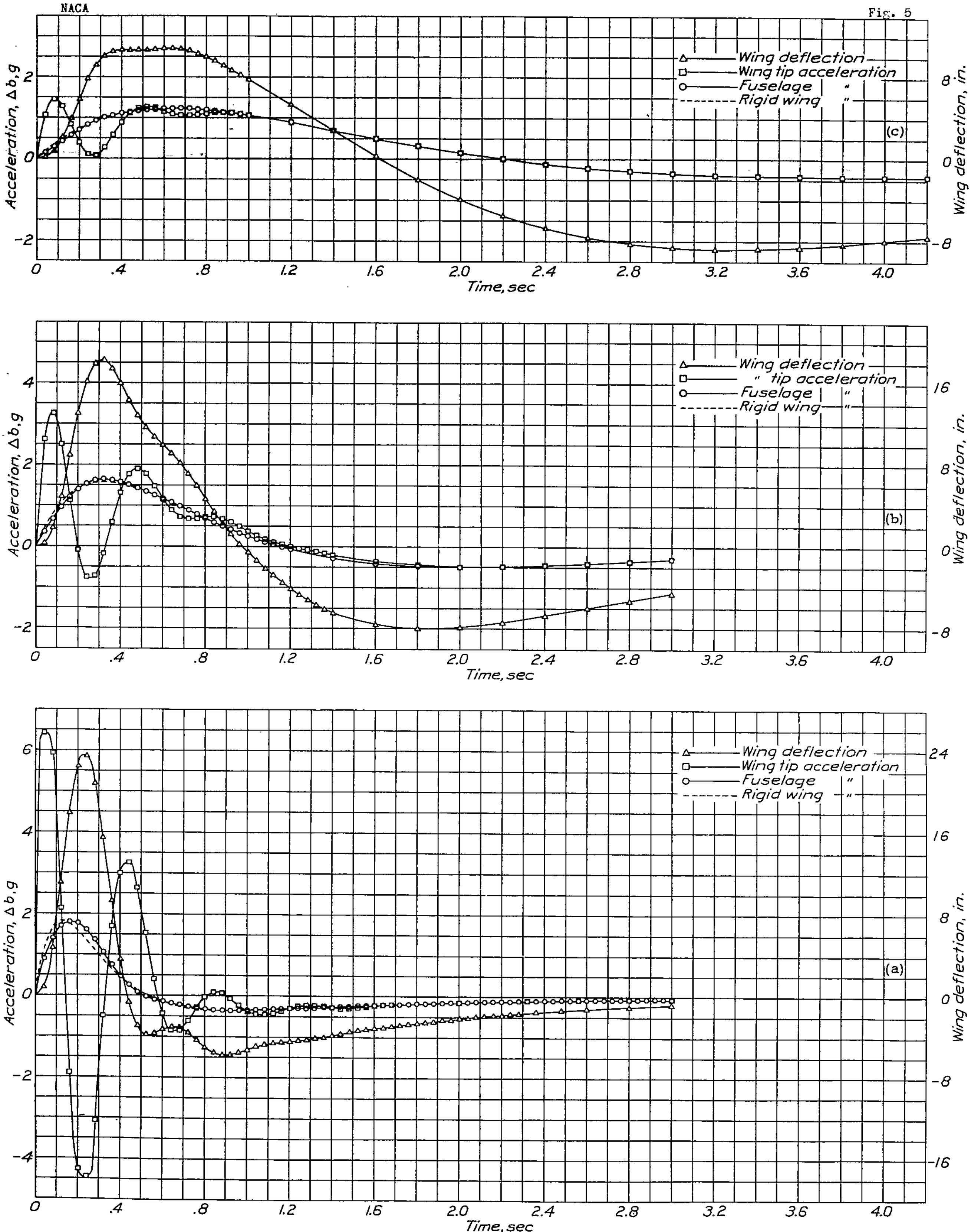

(a) $H_1$ = 4.26 chords; b = 4.27. (b) $H_1$ = 10.25 chords; b = 1.26. (c) $H_1$ = 19.53 chords; b = 0.459.

Figure 3.- History of reactions in a single gust. Model A, condition I.




(a) $H_1$ = 3.96 chords;b = 3.91. (b) $H_1$ = 10.08 chords;b = 1.30. (c) $H_1$ = 20.15 chords;b = 0.494.

Figure 4.- History of reactions in a single gust. Model A,condition II.




(a) $H_1$ = 3.75 chords;b = 6.94. (b) $H_1$ = 9.99 chords;b = 2.31. (c) $H_1$ = 19.98 chords;b = 0.887.
Figure 5.- History of reactions in a single gust. Model B,condition I.

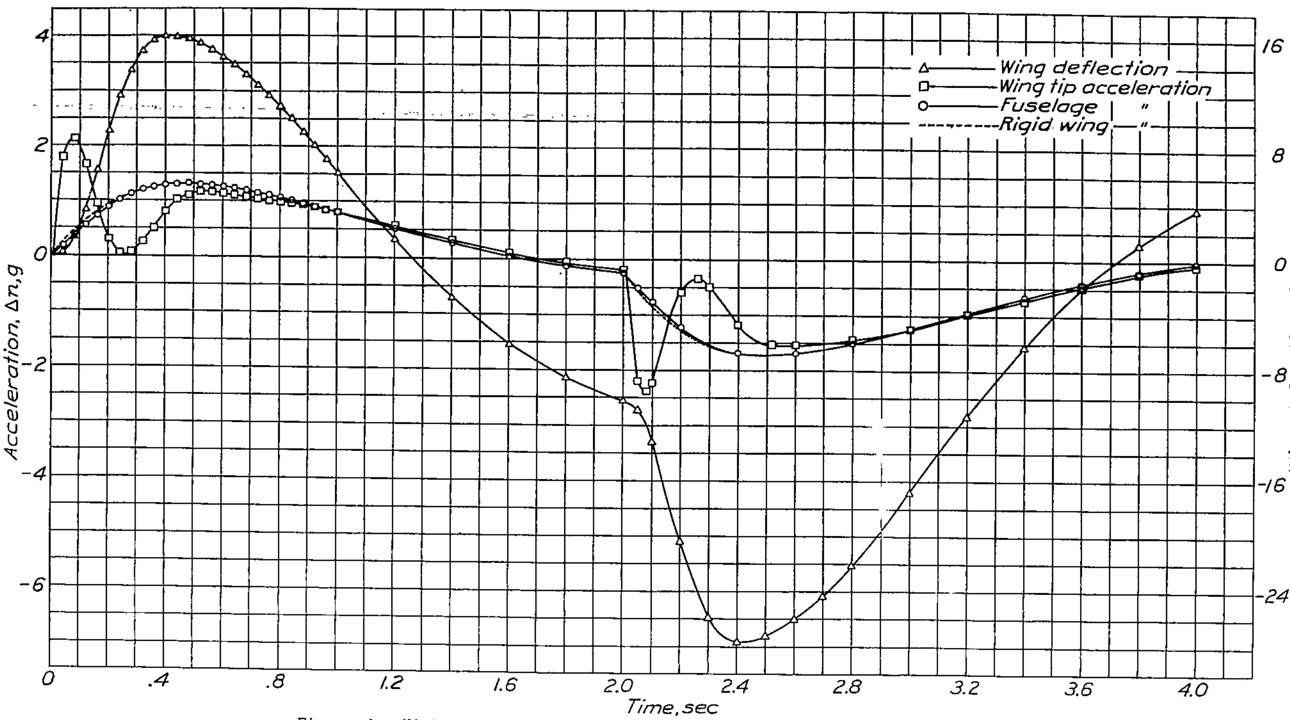



Figure 6.- History of reactions in a repeated gust. Model A, condition I, $H_1 = H_2 = 10.25$ chords, $H_3 = 32.48$ chords, $U_2 = -U_1$.

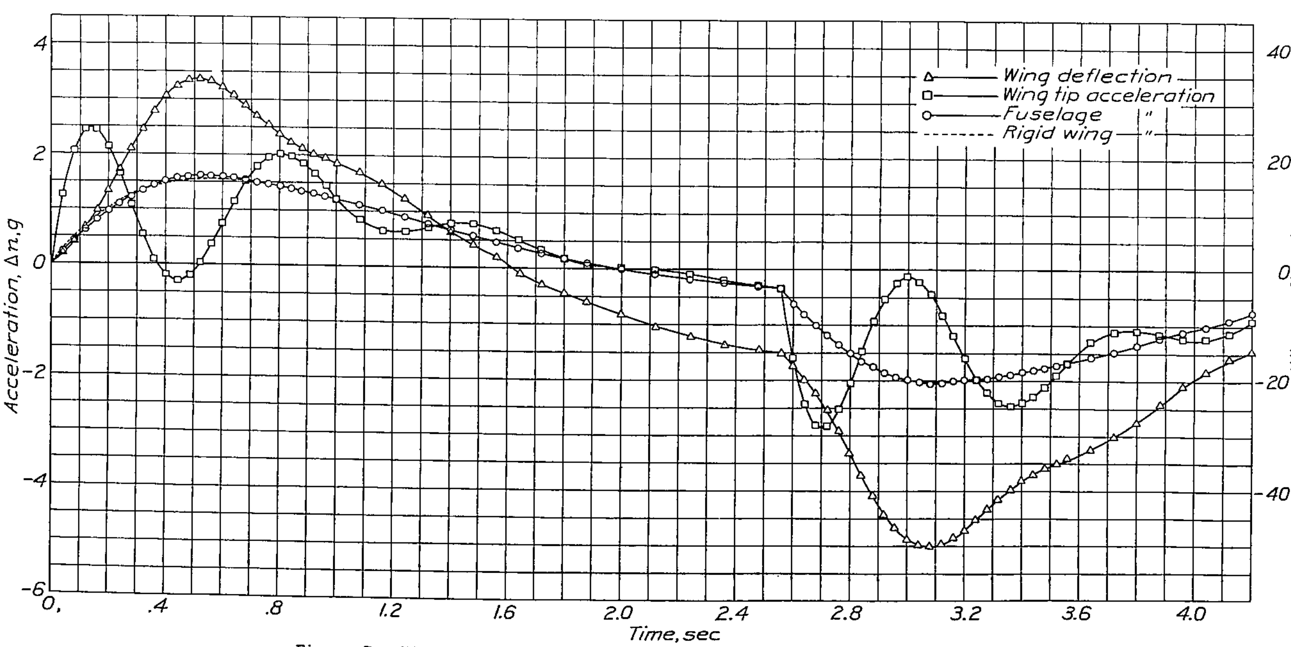



Figure 7.- History of reactions in a repeated gust. Model A; condition II, $H_1 = H_2 = 10.08$ chords, $H_3 = 33.83$ chords, $U_2 = -U_1$.

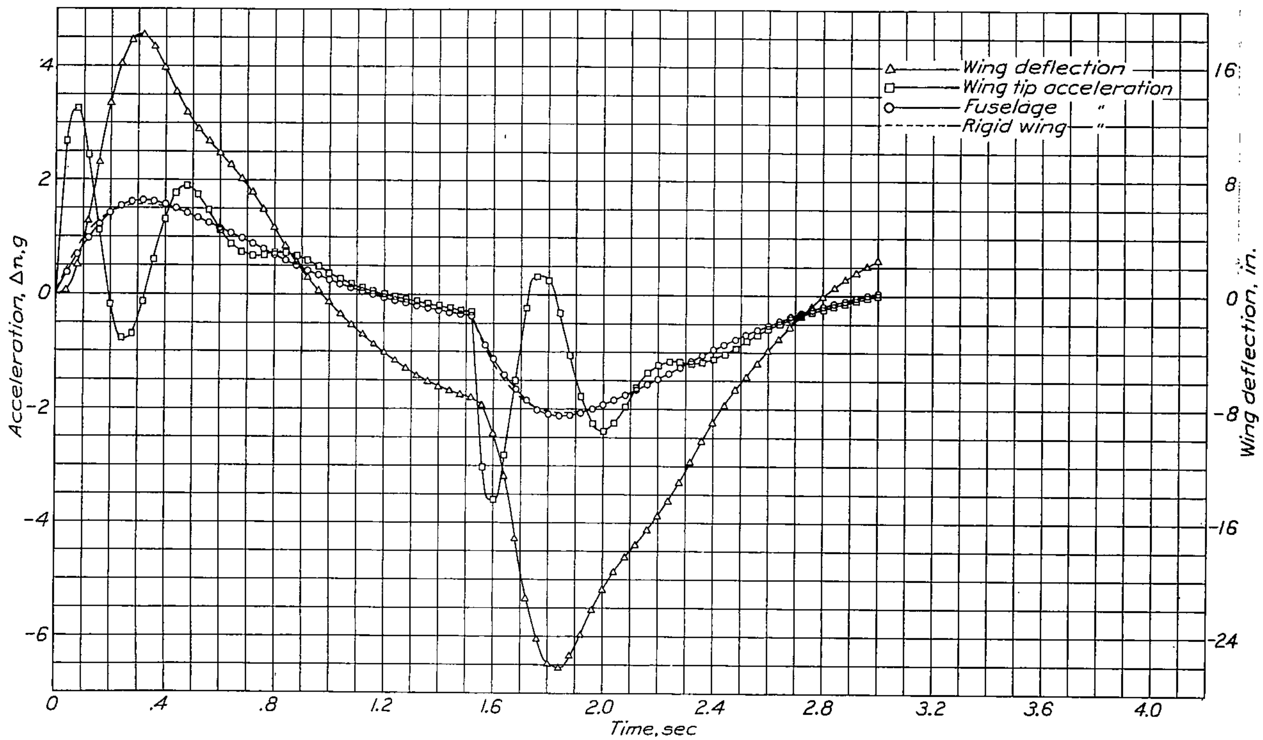
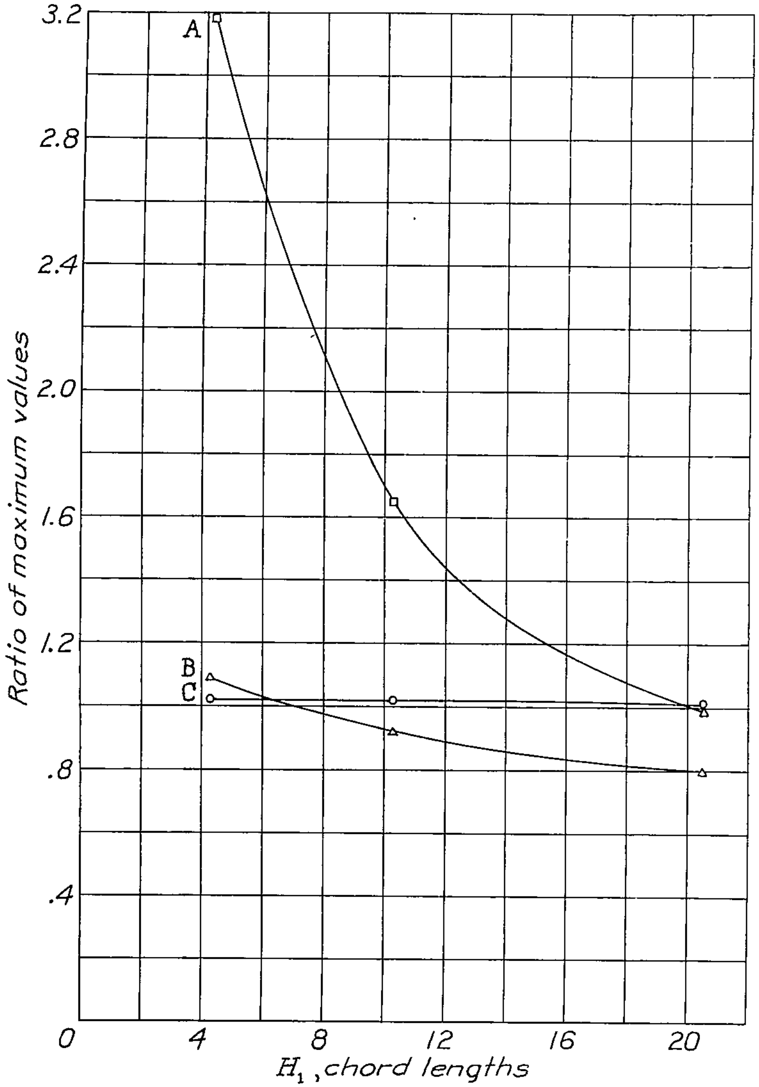
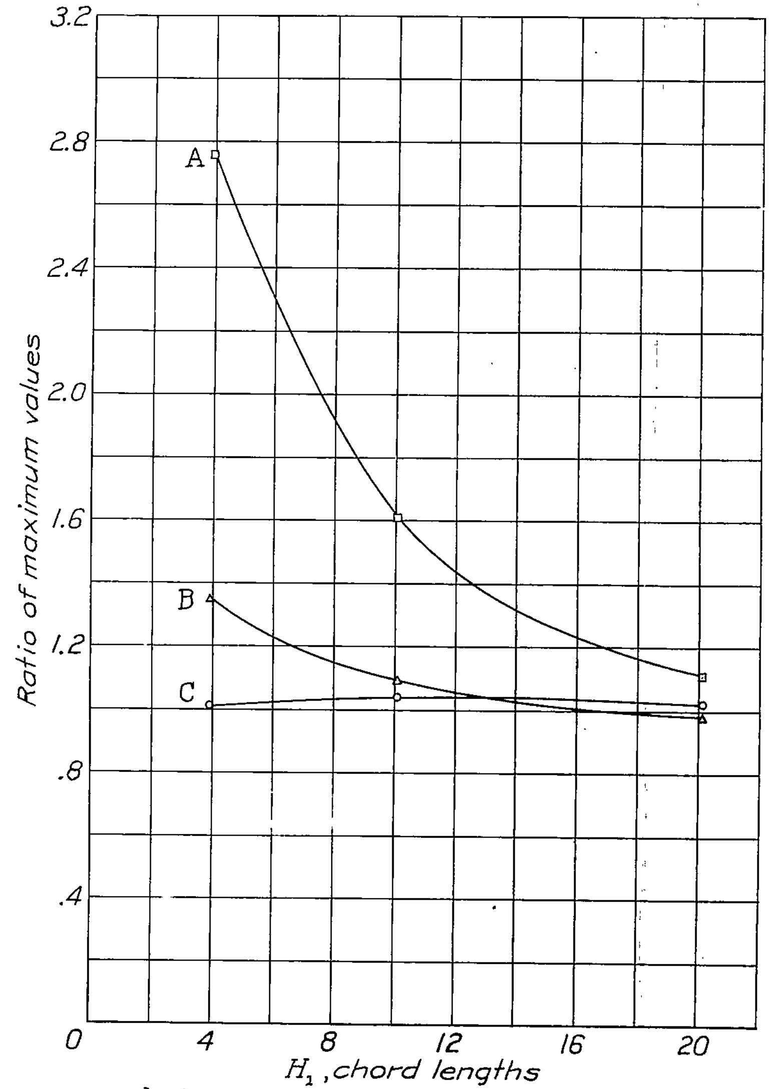



Figure 8.- History of reactions in a repeated gust. Model 3, condition I, $H_1 = H_2 = 9.99$ chords, $H_3 = 37.46$ chords, $U_2 = -U_1$.



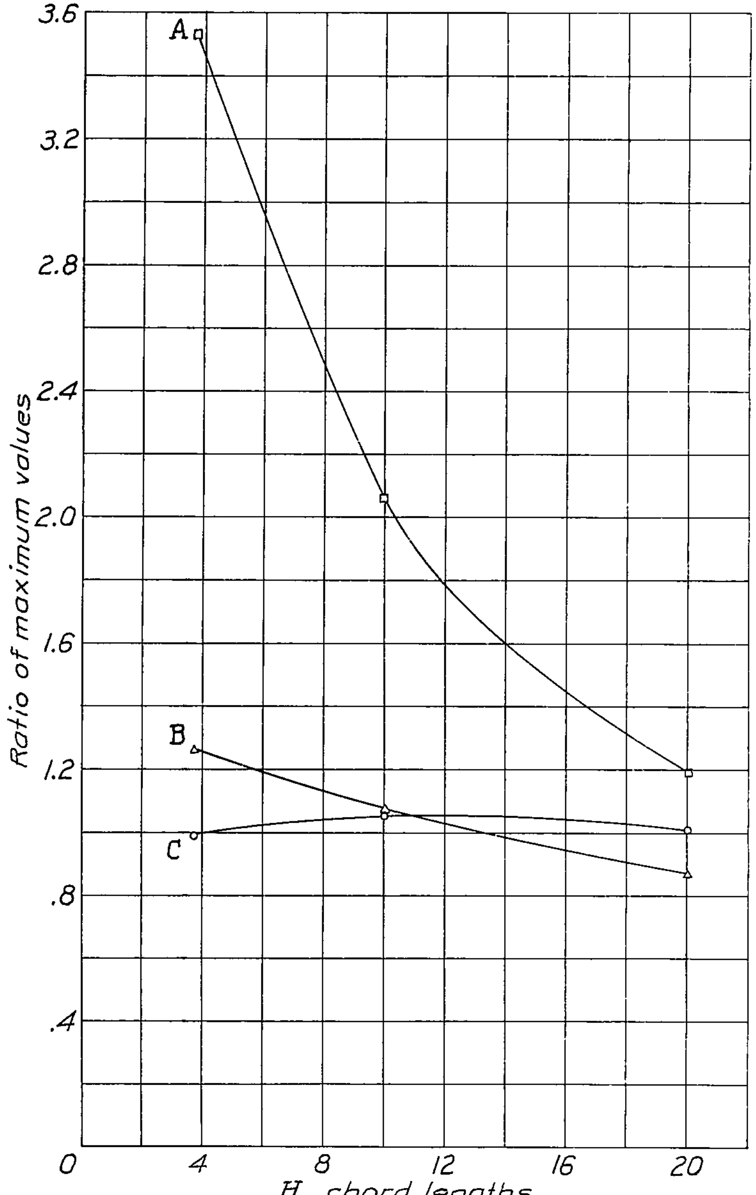

Curve A, $D^2\delta_w/\Delta n_r$ Curve B, $\delta_d/\delta_{st}$ Curve C, $D^2\delta_f/\Delta n_r$

Figure 9.- Variation of ratios of maximum dynamic to static reactions with gradient distance, Model A, condition I.




Curve A, $D^2\delta_w/\Delta n_r$ Curve B, $\delta_d/\delta_{st}$ Curve C, $D^2\delta_f/\Delta n_r$

Figure 10.- Variation of ratios of maximum dynamic to static reactions with gradient distance. Model A, condition II



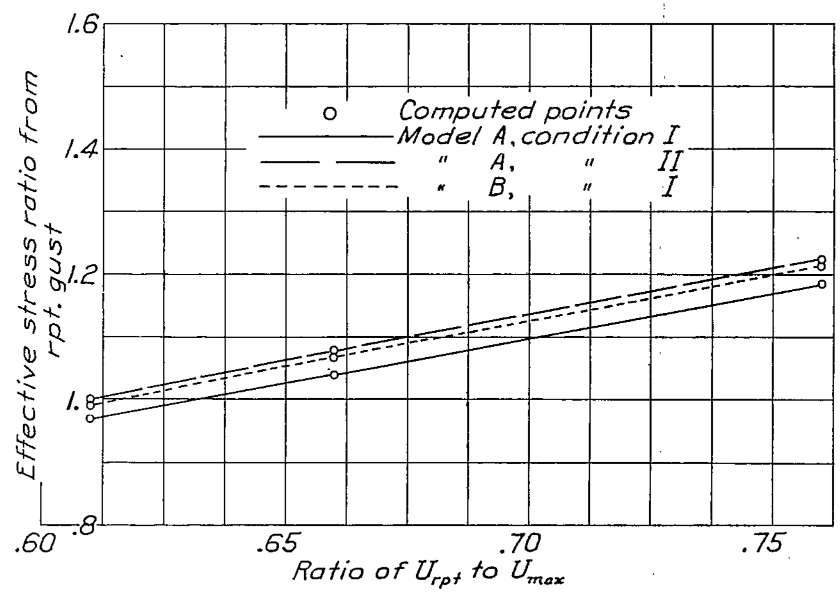




Figure 12.- Variation of effective stress ratio with ratio of $U_{rpt}$ to $U_{max}$.

Curve A, $D^2\delta_w/\Delta n_r$ Curve B, $\delta_d/\delta_{st}$ Curve C, $D^2\delta_f/\Delta n_r$

Figure 11.- Variation of ratios of maximum dynamic to static reactions with gradient distance. Model B, condition I.